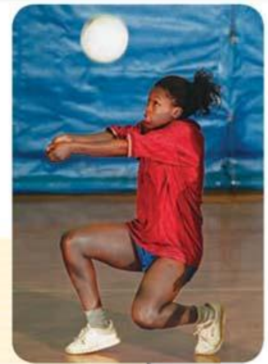
7.2 Use the Converse of the Pythagorean Theorem



Before You used the Pythagorean Theorem to find missing side lengths.
Now You will use its converse to determine if a triangle is a right triangle.
Why? So you can determine if a volleyball net is set up correctly, as in Ex. 38.

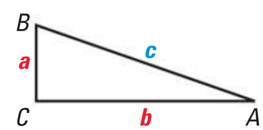
Key Vocabulary
- **acute triangle,** *p. 217*
- **obtuse triangle,** *p. 217*

The converse of the Pythagorean Theorem is also true. You can use it to verify that a triangle with given side lengths is a right triangle.

THEOREM *For Your Notebook*

THEOREM 7.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



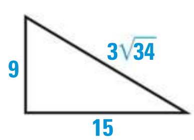
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Proof: Ex. 42, p. 446

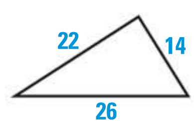
EXAMPLE 1 Verify right triangles

Tell whether the given triangle is a right triangle.

a.



b.



Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(3\sqrt{34})^2 \stackrel{?}{=} 9^2 + 15^2$

$9 \cdot 34 \stackrel{?}{=} 81 + 225$

$306 = 306$ ✓

The triangle is a right triangle.

b. $26^2 \stackrel{?}{=} 22^2 + 14^2$

$676 \stackrel{?}{=} 484 + 196$

$676 \neq 680$

The triangle is not a right triangle.

REVIEW ALGEBRA
Use a square root table or a calculator to find the decimal representation. So, $3\sqrt{34} \approx 17.493$ is the length of the longest side in part (a).

✓ **GUIDED PRACTICE for Example 1**

Tell whether a triangle with the given side lengths is a right triangle.

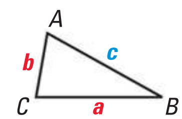
1. $4, 4\sqrt{3}, 8$ **2.** 10, 11, and 14 **3.** 5, 6, and $\sqrt{61}$

CLASSIFYING TRIANGLES The Converse of the Pythagorean Theorem is used to verify that a given triangle is a right triangle. The theorems below are used to verify that a given triangle is acute or obtuse.

THEOREMS — *For Your Notebook*

THEOREM 7.3

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an acute triangle.

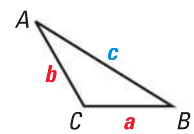


If $c^2 < a^2 + b^2$, then the triangle *ABC* is acute.

Proof: Ex. 40, p. 446

THEOREM 7.4

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an obtuse triangle.



If $c^2 > a^2 + b^2$, then triangle *ABC* is obtuse.

Proof: Ex. 41, p. 446

EXAMPLE 2 Classify triangles

Can segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle? If so, would the triangle be *acute, right,* or *obtuse*?

APPLY THEOREMS
The Triangle Inequality Theorem on page 330 states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Solution

STEP 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

$4.3 + 5.2 = 9.5$ $4.3 + 6.1 = 10.4$ $5.2 + 6.1 = 11.3$

$9.5 > 6.1$ $10.4 > 5.2$ $11.3 > 4.3$

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet can form a triangle.

STEP 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

$c^2 \overset{?}{\_\_} a^2 + b^2$ — **Compare c^2 with $a^2 + b^2$.**

$6.1^2 \overset{?}{\_\_} 4.3^2 + 5.2^2$ — **Substitute.**

$37.21 \overset{?}{\_\_} 18.49 + 27.04$ — **Simplify.**

$37.21 < 45.53$ — **c^2 is less than $a^2 + b^2$.**

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet form an acute triangle.

Animated Geometry at classzone.com

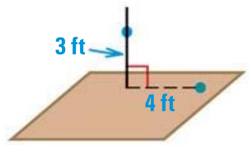
EXAMPLE 3 Use the Converse of the Pythagorean Theorem

CATAMARAN You are part of a crew that is installing the mast on a catamaran. When the mast is fastened properly, it is perpendicular to the trampoline deck. How can you check that the mast is perpendicular using a tape measure?

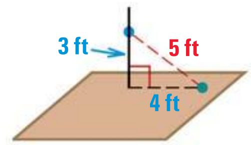
Solution

To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.

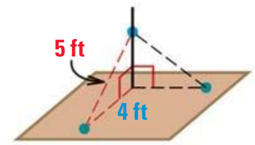
Think of the mast as a line and the deck as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the mast is perpendicular to different lines on the deck.



First place a mark 3 feet up the mast and a mark on the deck 4 feet from the mast.



Use the tape measure to check that the distance between the two marks is 5 feet. The mast makes a right angle with the line on the deck.



Finally, repeat the procedure to show that the mast is perpendicular to another line on the deck.

✓ GUIDED PRACTICE for Example 2 and 3

4. Show that segments with lengths 3, 4, and 6 can form a triangle and classify the triangle as *acute, right,* or *obtuse*.

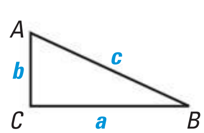
5. **WHAT IF?** In Example 3, could you use triangles with side lengths 2, 3, and 4 to verify that you have perpendicular lines? *Explain*.

CLASSIFYING TRIANGLES You can use the theorems from this lesson to classify a triangle as acute, right, or obtuse based on its side lengths.

CONCEPT SUMMARY *For Your Notebook*

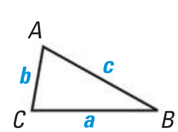
Methods for Classifying a Triangle by Angles Using its Side Lengths

Theorem 7.2



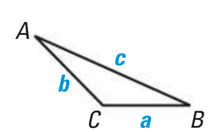
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.

Theorem 7.3



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

Theorem 7.4



If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

7.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 24, 25, 32, 38, 39, and 43

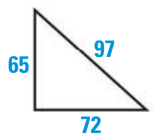
SKILL PRACTICE

1. **VOCABULARY** What is the longest side of a right triangle called?

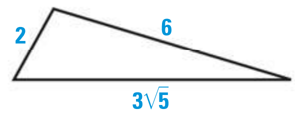
2. ★ **WRITING** *Explain* how the side lengths of a triangle can be used to classify it as acute, right, or obtuse.

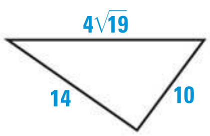
EXAMPLE 1 on p. 441 for Exs. 3–14

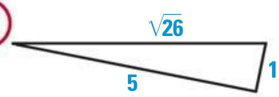
VERIFYING RIGHT TRIANGLES Tell whether the triangle is a right triangle.

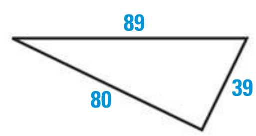
3.


4.


5.


6.


7.


8.


VERIFYING RIGHT TRIANGLES Tell whether the given side lengths of a triangle can represent a right triangle.

9. 9, 12, and 15
10. 9, 10, and 15
11. 36, 48, and 60
12. 6, 10, and $2\sqrt{34}$
13. 7, 14, and $7\sqrt{5}$
14. 10, 12, and 20

EXAMPLE 2 on p. 442 for Exs. 15–23

CLASSIFYING TRIANGLES In Exercises 15–23, decide if the segment lengths form a triangle. If so, would the triangle be *acute, right,* or *obtuse?*

15. 10, 11, and 14
16. 10, 15, and $5\sqrt{13}$
17. 24, 30, and $6\sqrt{43}$
18. 5, 6, and 7
19. 12, 16, and 20
20. 8, 10, and 12
21. 15, 20, and 36
22. 6, 8, and 10
23. 8.2, 4.1, and 12.2

24. ★ **MULTIPLE CHOICE** Which side lengths do not form a right triangle?

Ⓐ 5, 12, 13 Ⓑ 10, 24, 28 Ⓒ 15, 36, 39 Ⓓ 50, 120, 130

25. ★ **MULTIPLE CHOICE** What type of triangle has side lengths of 4, 7, and 9?

Ⓐ Acute scalene Ⓑ Right scalene
Ⓒ Obtuse scalene Ⓓ None of the above

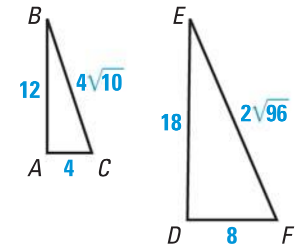
26. **ERROR ANALYSIS** A student tells you that if you double all the sides of a right triangle, the new triangle is obtuse. *Explain* why this statement is incorrect.

GRAPHING TRIANGLES Graph points *A*, *B*, and *C*. Connect the points to form $\triangle ABC$. Decide whether $\triangle ABC$ is *acute, right,* or *obtuse*.

27. $A(-2, 4)$, $B(6, 0)$, $C(-5, -2)$
28. $A(0, 2)$, $B(5, 1)$, $C(1, -1)$

29. **ALGEBRA** Tell whether a triangle with side lengths $5x$, $12x$, and $13x$ (where $x > 0$) is *acute*, *right*, or *obtuse*.

USING DIAGRAMS In Exercises 30 and 31, copy and complete the statement with <, >, or =, if possible. If it is not possible, *explain* why.



30. $m\angle A$ __?__ $m\angle D$

31. $m\angle B + m\angle C$ __?__ $m\angle E + m\angle F$

32. ★ **OPEN-ENDED MATH** The side lengths of a triangle are 6, 8, and x (where $x > 0$). What are the values of x that make the triangle a right triangle? an acute triangle? an obtuse triangle?

33. **ALGEBRA** The sides of a triangle have lengths x, $x + 4$, and 20. If the length of the longest side is 20, what values of x make the triangle acute?

34. **CHALLENGE** The sides of a triangle have lengths $4x + 6$, $2x + 1$, and $6x - 1$. If the length of the longest side is $6x - 1$, what values of x make the triangle obtuse?

PROBLEM SOLVING

EXAMPLE 3
on p. 443
for Ex. 35

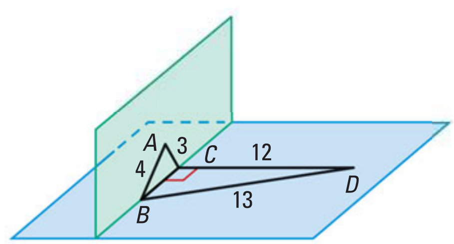
35. **PAINTING** You are making a canvas frame for a painting using stretcher bars. The rectangular painting will be 10 inches long and 8 inches wide. Using a ruler, how can you be certain that the corners of the frame are 90°?

@HomeTutor for problem solving help at classzone.com

36. **WALKING** You walk 749 feet due east to the gym from your home. From the gym you walk 800 feet southwest to the library. Finally, you walk 305 feet from the library back home. Do you live directly north of the library? *Explain*.

@HomeTutor for problem solving help at classzone.com

37. **MULTI-STEP PROBLEM** Use the diagram shown.



a. Find BC.

b. Use the Converse of the Pythagorean Theorem to show that $\triangle ABC$ is a right triangle.

c. Draw and label a similar diagram where $\triangle DBC$ remains a right triangle, but $\triangle ABC$ is not.

38. ★ **SHORT RESPONSE** You are setting up a volleyball net. To stabilize the pole, you tie one end of a rope to the pole 7 feet from the ground. You tie the other end of the rope to a stake that is 4 feet from the pole. The rope between the pole and stake is about 8 feet 4 inches long. Is the pole perpendicular to the ground? *Explain*. If it is not, how can you fix it?

39. ★ **EXTENDED RESPONSE** You are considering buying a used car. You would like to know whether the frame is sound. A sound frame of the car should be rectangular, so it has four right angles. You plan to measure the shadow of the car on the ground as the sun shines directly on the car.

a. You make a triangle with three tape measures on one corner. It has side lengths 12 inches, 16 inches, and 20 inches. Is this a right triangle? *Explain*.

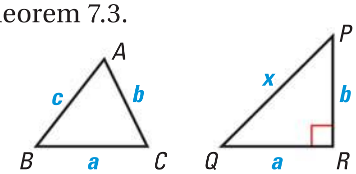
b. You make a triangle on a second corner with side lengths 9 inches, 12 inches, and 18 inches. Is this a right triangle? *Explain*.

c. The car owner says the car was never in an accident. Do you believe this claim? *Explain*.

40. **PROVING THEOREM 7.3** Copy and complete the proof of Theorem 7.3.

GIVEN ▶ In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side.

PROVE ▶ $\triangle ABC$ is an acute triangle.



Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x, where $\angle R$ is a right angle and x is the length of the longest side. Compare lengths c and x.

STATEMENTS	REASONS
1. In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	**1.** ?
2. $a^2 + b^2 = x^2$	**2.** ?
3. $c^2 < x^2$	**3.** ?
4. $c < x$	**4.** A property of square roots
5. $m\angle R = 90°$	**5.** ?
6. $m\angle C < m\angle$?	**6.** Converse of the Hinge Theorem
7. $m\angle C < 90°$	**7.** ?
8. $\angle C$ is an acute angle.	**8.** ?
9. $\triangle ABC$ is an acute triangle.	**9.** ?

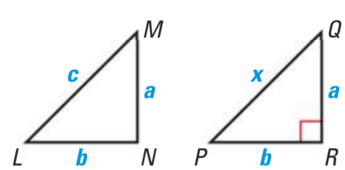
41. **PROVING THEOREM 7.4** Prove Theorem 7.4. Include a diagram and GIVEN and PROVE statements. (*Hint*: Look back at Exercise 40.)

42. **PROVING THEOREM 7.2** Prove the Converse of the Pythagorean Theorem.

GIVEN ▶ In $\triangle LMN$, $\overline{LM}$ is the longest side, and $c^2 = a^2 + b^2$.

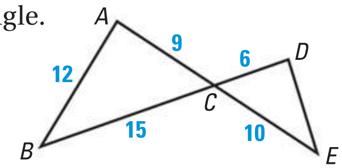
PROVE ▶ $\triangle LMN$ is a right triangle.

Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x. Compare lengths c and x.



★ = STANDARDIZED TEST PRACTICE

43. ★ **SHORT RESPONSE** *Explain* why $\angle D$ must be a right angle.



44. **COORDINATE PLANE** Use graph paper.

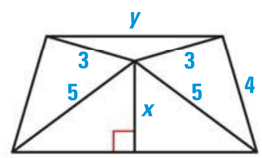
a. Graph $\triangle ABC$ with $A(-7, 2)$, $B(0, 1)$ and $C(-4, 4)$.

b. Use the slopes of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

c. Use the lengths of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

d. Did you get the same answer in parts (b) and (c)? If not, *explain* why.

45. **CHALLENGE** Find the values of x and y.

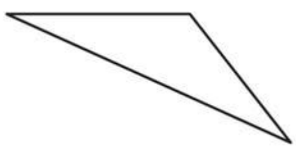


MIXED REVIEW

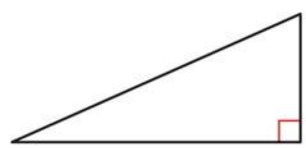
PREVIEW
Prepare for Lesson 7.3 in Exs. 46–48.

In Exercises 46–48, copy the triangle and draw one of its altitudes. *(p. 319)*

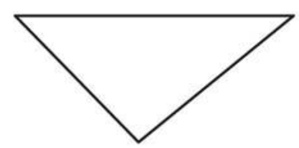
46.



47.



48.



Copy and complete the statement. *(p. 364)*

49. If $\frac{10}{x} = \frac{7}{y}$, then $\frac{10}{7} = \frac{?}{?}$.

50. If $\frac{x}{15} = \frac{y}{2}$, then $\frac{x}{y} = \frac{?}{?}$.

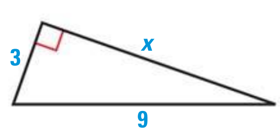
51. If $\frac{x}{8} = \frac{y}{9}$, then $\frac{x+8}{8} = \frac{?}{?}$.

52. The perimeter of a rectangle is 135 feet. The ratio of the length to the width is 8 : 1. Find the length and the width. *(p. 372)*

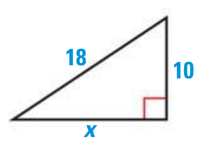
QUIZ *for Lessons 7.1–7.2*

Find the unknown side length. Write your answer in simplest radical form. *(p. 433)*

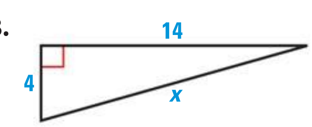
1.



2.



3.



Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*. *(p. 441)*

4. 6, 7, and 9

5. 10, 12, and 16

6. 8, 16, and $8\sqrt{6}$

7. 20, 21, and 29

8. 8, 3, $\sqrt{73}$

9. 8, 10, and 12